MA1161
Test 3
Spring 2007

Name:_____

Instructions: **You may use your calculator on the entire test. However, if you use a trial-and-error or guess-and-check method, or read a solution from your calculator when a calculus or algebraic method is available, you will not receive full credit. You must show enough work to justify all answers.**

Page	Score	
1		/ 28
2		/ 17
3		/ 10
4		/ 15
5		/ 15
6		/ 15
Final Score		/ 60

1. Let $h(x) = g(f(x))$. Also, let the following be true: $f(2) = 6$, $f'(2) = \frac{1}{3}$, $g(6) = 7$, and $g'(6) = 3$. Find $h'(2)$.

$h'(2) = 1$ [7]

2. Differentiate the following:

   (a) $f(x) = (2x^3 + e^{2x})^3$

$f'(x) = 3(2x^3 + e^{2x})^2(6x^2 + 2e^{2x})$ [7]

   (b) $g(\theta) = \theta \sin(\theta^2)$

$g'(\theta) = 2\theta^2 \cos(\theta^2) + \sin(\theta^2)$ [7]

   (c) $h(x) = \arctan(\sqrt{x})$

$h'(x) = \frac{1}{2\sqrt{x}(1+x)}$ [7]

3. Let $y = e^{(-x^2+1)}$. Find y''.

$y'' = e^{(-x^2+1)}(4x^2 - 2)$ [12]

4. Let $f(x) = \frac{1}{4}x^4 - x^2$.

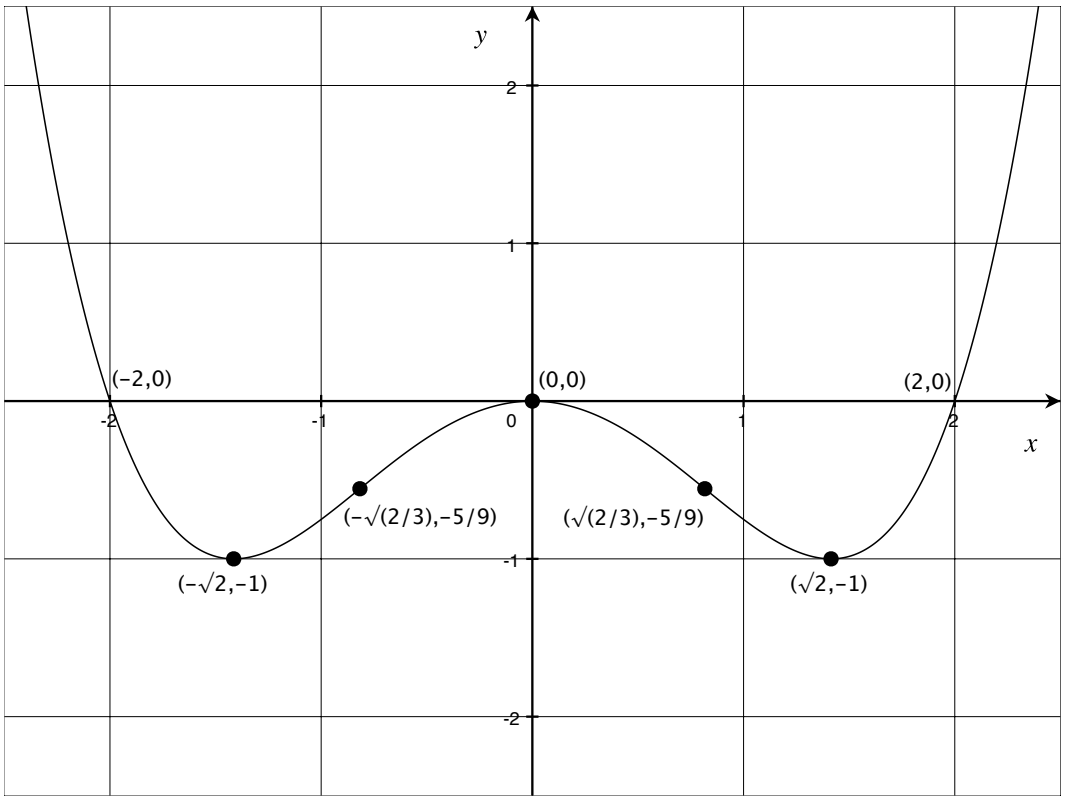
   (a) Find the critical points for this function. For each, determine whether it is a local minimum, local maximum or neither. [5]

Solution.
Critical points are $-\sqrt{2}, 0$, and $\sqrt{2}$.
$-\sqrt{2}$ and $\sqrt{2}$ are local minimums.
0 is a local maximum. □

(b) Identify the intervals on which $f(x)$ is concave up and concave down. [4]

Solution.
Concave up: $\left(-\infty, -\sqrt{\frac{2}{3}}\right) \cup \left(\sqrt{\frac{2}{3}}, \infty\right)$
Concave down: $\left(-\sqrt{\frac{2}{3}}, \sqrt{\frac{2}{3}}\right)$ □

(c) Graph $f(x)$. Be sure to include BOTH coordinates of all local minima and maxima, inflection points, x-intercept(s) and y-intercept. [6]



5. Use implicit differentiation to find the equation of a line tangent to the following curve at the point $(0, 3)$.

$$y^3 + x^2y + x^2 - 3y^2 = 0$$

$y = 3$ [15]

6. A rectangular poster is to have a total area of 2700 cm^2 with 3 cm margins at the bottom and sides and a 5 cm margin at the top. What dimensions will give the largest printed area (the area inside the margins)? Be sure to justify that your answer is truly a maximum.

60 cm tall by 45 cm wide [15]

7. A 4 meter ladder stands against a high wall. The foot of the ladder moves outward at a speed of 0.2 m/s when the foot is 1 meter from the wall. At that moment, how fast is the top of the ladder falling?

0.05164 m/s [15]